

BOWED STRING SIMULATION USING AN ELASTO-PLASTIC FRICTION MODEL

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ABSTRACT

This paper proposes a digital waveguide model of a bowed string in which the interaction between the bow and the string is modeled using an elasto-plastic friction model. The interaction model is presented, and its advantages with respect to usual static models are discussed. Details about the numerical implementation are addressed. Numerical simulations show that a hysteresis loop in the friction versus velocity plane is created, which is qualitatively compatible with experiments on real instruments and with the behavior of a recently proposed thermodynamical friction model applied to bowed string simulations.

1. INTRODUCTION

The bow-string interaction is a complex phenomenon that belongs to the larger field of frictional induced vibrations. The most widely accepted theoretical models of the bow-string interaction assume that the excitation ideally occurs at a single point and that the frictional force at the contact point is a function of the relative sliding velocity between the bow and the string (see for example [1, 2]). Many relevant features of the bowed string motion are successfully explained by these *static* models.

Recently, Smith and Woodhouse [3] proposed a more sophisticated model, based on the observation that the interfacial rosin layer exhibits plastic deformation at the contact point due to fast temperature changes within one oscillation period. For this reason, they proposed a model called *plastic*, in which friction depends on the “thermal history” around the contact zone. As a consequence, the model exhibits an hysteresis loop in the force versus velocity plane.

Outside musical acoustics studies, a plethora of friction models are available from the literature of robotics and haptics, as well as automatic control. *Dynamic* models have been proposed, in which the dependence of friction on the relative sliding velocity is given by a differential equation [4, 5, 6, 7], which is meant to account for transient behaviors.

In this paper we apply a class of *elasto-plastic* dynamic friction models to the simulation of a bow exciting a string. An earlier attempt to use this model in the context of bowed string simulation has been already proposed in [8]. However, the numerical implementation used by the authors does not allow accurate simulations of the model, and introduces instability problems. The numerical scheme adopted here provides an accurate yet efficient numerical implementation of the non-linear equations. The very same numerical model has been applied elsewhere to simulate frictional

interaction between modal resonators [9], and applied to the design of “everyday” (non-musical) sounds [10].

Section 2 describes the interaction model and compares it with the plastic approach proposed by Smith and Woodhouse. Section 3 presents numerical simulations from both models, and shows that they provide qualitatively similar behaviors. Ideas for future work are sketched in the concluding section 4.

2. DYNAMIC FRICTION MODELS

In order to adequately describe friction phenomena at low velocities (i.e., stick-slip motion, presliding behavior, frictional memory, etc.) researchers have recently developed dynamic models that describe the dependence of friction on the relative velocity between two contacting bodies through a differential equation. Dynamic models are able to take into account presliding behavior for very small displacements, where the friction force increases gradually with the displacement.

The first attempt to describe presliding is due to Dahl (see [4] for a review): the Dahl model successfully simulates presliding displacement, but does not account for the sliding regime and related features (e.g., the Stribeck effect, i.e. the dip of the force at low relative velocities). The LuGre model [5] extended Dahl’s work in order to include such effects. However, it was shown that this model exhibits drift for arbitrarily small external forces, which is not physically consistent. This effect has been explained in [7] by observing that LuGre does not allow purely elastic regime: therefore, a class of *elasto-plastic* models has been proposed in [7], where the drawbacks of LuGre are overcome.

Before addressing the elasto-plastic approach in section 2.2, we briefly review in section 2.1 the plastic model by Smith and Woodhouse.

2.1. The plastic friction model

Experimental results reported by Smith and Woodhouse [3] show that, in stick-slip conditions, the measured friction force versus the sliding speed exhibits large hysteresis loops, i.e. the instantaneous sliding speed does not determine the friction force during the oscillation. This hysteresis loop suggests that the contact force has a dependence on some additional variables, and that the system state must be enlarged in order to account for this phenomenon.

In [3], the authors argued that the contact temperature during the oscillation cycle plays an important role by affecting the coefficient of friction of the interfacial rosin layer. They developed a method for measuring the varying coefficient of friction during

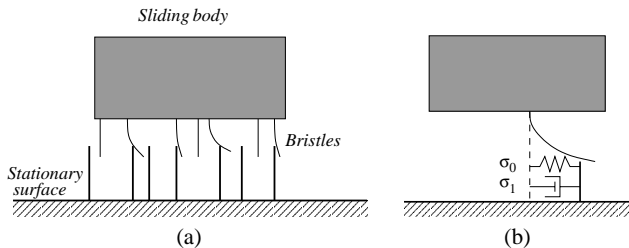


Figure 1: (a) bristle interpretation of the friction mechanism; (b) the LuGre model as an averaged description of the bristle behavior.

individual cycles of stick-slip vibrations, and proposed a model that includes this information. The authors called this model *plastic*, since it accounts for plastic deformations of the rosin layer according to temperature variations.

In this model, friction is therefore given by:

$$f = \text{sgn}(v)F_N \cdot \mu(T), \quad (1)$$

where F_N is the normal bow force, $\mu(T)$ is a temperature dependent friction coefficient as described in [3], and v is the relative velocity between the bow and the string.

It is shown in [3] that this model reproduces the experimentally observed hysteresis loops in the force versus velocity plane.

2.2. The elasto-plastic model

Let us consider two facing surfaces in frictional contact. The friction between the two surfaces can be interpreted to be caused by a large number of bristles, each contributing a fraction of the total friction load, as shown in figure 1(a). The load contributed by each bristle is proportional to the strain of the bristle. When the strain exceeds a certain level the bond is broken.

The LuGre model [5] makes use of this interpretation, and describes the deflection of the bristles in an averaged way through a single degree of freedom z . When a tangential force is applied, the bristles are assumed to be subject to restoring elastic and dissipative forces (see figure 1(b)). If the deflection is large enough, the bristles start to slip.

Denoting by z the average bristle deflection, the model is given by:

$$\begin{aligned} \dot{z}(v, z) &= v - \sigma_0 \frac{|v|}{g(v)} z, \\ F &= \sigma_0 z + \sigma_1(v) \dot{z} + f(v), \end{aligned} \quad (2)$$

where σ_0 is the stiffness of the bristles, and $\sigma_1(v)$ is the damping. The function $g(v)$ describes the steady state displacement for constant sliding velocities ($\dot{z} = 0$, then $z = 1/[\sigma_0 g(v)]$).

The physical interpretation of this model is as follows: contact surfaces are very irregular at microscopic level, and we visualize this as two rigid bodies that make contact through elastic bristles. When a tangential force is applied, the bristles will deflect like springs and dampers which give rise to the friction force. The average deflection of the bristles corresponds to the internal state of the dynamic friction model z .

One drawback of the LuGre model is that it exhibits drift for arbitrarily small external forces: this is not a physically consistent behavior, and is rather due to inaccurate modeling of the presliding behavior. The *elasto-plastic* class of models [7] is also based

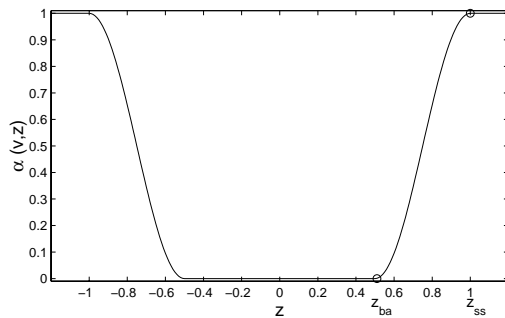


Figure 2: Smooth elastic-to-plastic transition provided by the function $\alpha(v, z)$.

on the bristle interpretation of friction, and overcomes the drawbacks of LuGre by using the following formulation for the bristle displacement:

$$\dot{z} = v \left[1 - \alpha(z, v) \frac{z}{z_{ss}(v)} \right]. \quad (3)$$

Compared to equation (2), the new ingredient is the function $\alpha(z, v)$. This is an adhesion map which controls the rate of change of z to avoid drift. Following Dupont *et al.* [7], we use a function $\alpha(z, v)$ parametrized as

$$\alpha(v, z) = \begin{cases} 0 & |z| < z_{ba}, \text{sgn}(v) = \text{sgn}(z) \\ \alpha_m(v, z) & z_{ba} < |z| < z_{ss}(v), \text{sgn}(v) = \text{sgn}(z) \\ 1 & |z| > z_{ss}(v), \text{sgn}(v) = \text{sgn}(z) \\ 0 & \text{sgn}(v) \neq \text{sgn}(z), \end{cases} \quad (4)$$

where z_{ba} is the breakaway displacement below which the presliding is purely elastic, and

$$\alpha_m(v, z) = \frac{1}{2} \left[1 + \sin \left(\pi \frac{z - \frac{1}{2}(z_{ss}(v) + z_{ba})}{z_{ss}(v) - z_{ba}} \right) \right] \quad (5)$$

ensures a smooth transition between elastic and plastic behavior (see figure 2). Indeed, for small displacements $\alpha = 0$ and consequently $\dot{z} = v$ (elastic presliding), while for larger displacements mixed elastic-plastic sliding is entered. Finally, at steady state purely plastic regime is achieved with $\alpha = 1$, $\dot{z} = 0$, and $z = z_{ss}$.

We use the following steady-state friction characteristic z_{ss} , defined as in [5]:

$$z_{ss}(v) = \frac{\text{sgn}(v)}{\sigma_0} \left[f_c + (f_s - f_c) e^{-(v/v_s)^2} \right], \quad (6)$$

where f_c, f_s are the Coulomb force and the stiction force respectively, while v_s is the Stribeck velocity.

Equations (6) and (4) are just two possible parametrizations for the functions $z_{ss}(v)$ and $\alpha(z, v)$. In practice, significant deviations from these curves can be tolerated without affecting the model behavior significantly.

3. APPLICATION TO BOWED STRING SIMULATIONS

In order to obtain a comparison with the plastic model proposed in [3], we applied our model to the simulation of a string excited by a bow.

3.1. Coupling exciter and resonator

The string resonator is modeled here using one dimensional digital waveguides while the bow is represented as an ideal lumped mass. The equations for the bow are discretized using the bilinear transformation, that corresponds to trapezoid integration of continuous-time functions [11]. It can be verified that the discrete-time relative velocity $v(n)$ is given by

$$v(n) = \tilde{v}(n) + \mathbf{k}(1)\dot{z}(n), \quad (7)$$

where \tilde{v} is a computable quantity (i.e. a linear combination of variables that are known at time n). The coefficient $\mathbf{k}(1)$ isolates the dependence of $v(n)$ upon $\dot{z}(n)$ at the current time instant n . The bristle equation of dynamics is also discretized using the bilinear transformation, with time step $T_s = 1/F_s$:

$$\begin{aligned} z(n) &\approx \underbrace{z(n-1) + \frac{T_s}{2}y(n-1) + \frac{T_s}{2}\dot{z}(n)}_{\tilde{z}(n)} + \mathbf{k}(2)\dot{z}(n) \\ &\triangleq \tilde{z}(n) + \mathbf{k}(2)\dot{z}(n) \end{aligned} \quad (8)$$

where $\mathbf{k}(2) = T_s/2$. Again, the term $\tilde{z}(n)$ represents a computable quantity.

Note that when the two interacting objects are coupled through the non-linear friction interaction, a delay-free path is generated in the computation: namely, the relative velocity $v(n)$ and the bristle displacement $z(n)$ depend on the term $\dot{z}(n)$, which in turn depends non-linearly on the pair $(v(n), z(n))$ through equation (3). This problem is well studied in the literature of physical modeling. In [8], the authors chose the same numerical implementation used in [6], which introduces one sample delay to solve computational problems. However, this solution is known to introduce inaccuracies and even instability in the simulations. Here we adopt the approach proposed in [12]: at each time step the value $\dot{z}(n)$ is found as a function of the pair (\tilde{v}, \tilde{z}) , and is computed using Newton-Raphson iterations by finding a local zero of the function $g(\dot{z}) = [\dot{z}(\tilde{v}(n) + \mathbf{k}(1)\dot{z}(n), \tilde{z}(n) + \mathbf{k}(2)\dot{z}(n)) - \dot{z}]$ (see also [10] for numerical details). Since simulations show that seven iterations of the Newton-Raphson algorithm typically allow to converge to the solution, the iterative zero-finding procedure is computationally efficient for real-time implementation.

3.2. Simulations

The control parameters that are typically used to drive a bowed string instrument are the bow velocity V_b and bow force F_N . In order to perform meaningful comparisons between the elasto-plastic and the plastic [3] models, the relationship between the low-level parameters of equations (3–6) and the pair (V_b, F_N) must be found.

As described in section 3.1, in our modeling approach the bow is treated as a lumped mass. As such, it can only be controlled by the user through external driving forces, and the bow velocity V_b can not be directly set. For this reason, the simulations described in this section have been obtained using a slightly different version of the model: in this formulation, no external forces are used and the state of the bow is explicitly updated at each time-step by forcing the bow velocity to assume the control value V_b .

The parameters (f_c, f_s) are related to F_N through the static and dynamic friction coefficients as $f_s = \mu_s F_N$ and $f_c = \mu_d F_N$. Reasonable values for the static and dynamic coefficients in violin bows are $\mu_s \approx 0.4 \dots 0.5$ and $\mu_d \approx 0.2$, respectively (see [13]).

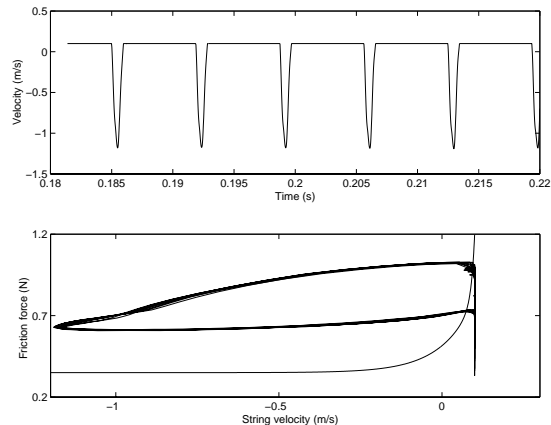


Figure 3: *Top: velocity at the bow point for a cello D string (147 Hz) bowed with $V_b = 0.1$ m/s and $F_N = 1.1$ N. Bottom: velocity versus coefficient of friction. Both plots are generated using the model proposed in [3]. Courtesy of Jim Woodhouse.*

The remaining parameters are taken from the literature. Note that σ_0 defines the “degree of dynamicity” of the dynamic model (for $\sigma_0 \rightarrow \infty$ the bristles do not move anymore) or, equivalently, the magnitude of the allowed presliding displacement, while σ_1 describes the internal dissipation of vibrating bristles.

Figure 3 shows the results of applying the plastic model [3] to the simulation of a cello *D* string, tuned to 147 Hz, with a Q factor of 500 and a stiffness coefficient $B = 0.0003$ N m². The upper plot of figure 3 shows the snapshot of the time-domain velocity waveform at the bowing point obtained during the steady state portion of the motion, for $V_b = 0.1$ m/s and $F_N = 1.1$ N. Note that the Helmholtz motion, i.e. the ideal motion of the bowed string, is achieved. The lower plot of figure 3 shows hysteresis loop in the friction versus velocity characteristics.

Figure 4 shows the same signals as in 3, obtained using the elasto-plastic model, with parameters $\sigma_0 = 6000$ and $\sigma_1 = 0$. Note that the behavior of the two models is qualitatively similar. The snapshot of the time-domain velocity waveform at the bowing point (upper plot) shows that, as before, the Helmholtz motion is achieved. More interestingly, the two models exhibit similar hysteretical behaviors of the friction versus velocity curve.

One noticeable difference between the two models is that the plastic model assumes an “ideal” ($v \equiv 0$) stick phase, which can be noticed in the horizontal segments of the upper plot in figure 3. On the other hand, the corresponding plot in figure 4 shows that the ideal condition $v \equiv 0$ is never achieved, and small oscillations occur even during the stick phase. This phenomenon is also noticed in the lower plot of figure 4, specifically in the small ripples around $v = 0$.

This transient behavior affects the sound quality dramatically. Figure 5 shows the result of elasto-plastic simulations with the same parameters as before, but with a larger value for the σ_1 coefficient. Note that the waveforms are similar. However, since the parameter σ_1 controls the internal bristle dissipation and consequently affects the transient behavior, the resulting micro-oscillations during the stick phase are also affected and the perceptual results are noticeably different.

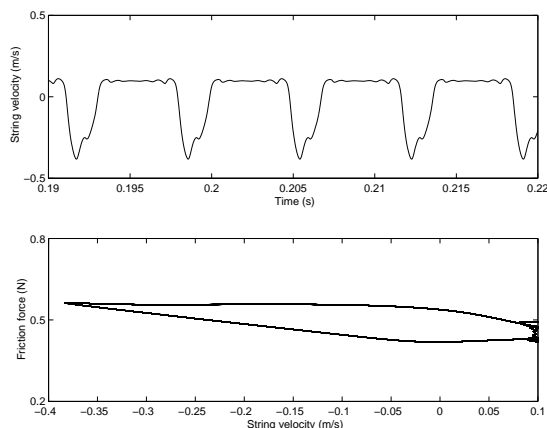


Figure 4: *Top: velocity at the bow point for a cello D string (147 Hz) bowed with $V_b = 0.1$ m/s and $F_N = 1.1$ N, $\sigma_0 = 6000$, $\sigma_1 = 0$. Bottom: velocity (m/s) versus friction force (N).*

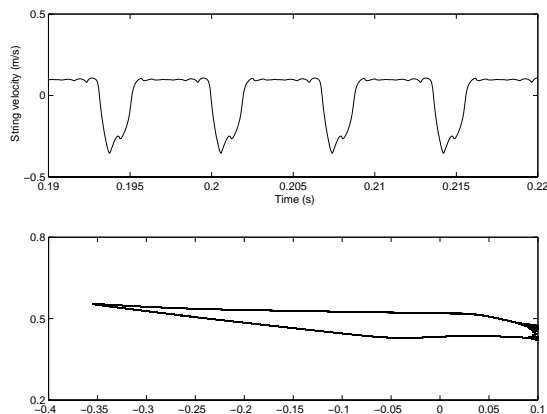


Figure 5: *Top: velocity at the bow point for a cello D string (147 Hz) bowed with $V_b = 0.1$ m/s and $F_N = 1.1$ N, $\sigma_0 = 6000$, $\sigma_1 = 0.5$. Bottom: velocity (m/s) versus friction force (N).*

4. DISCUSSION

We have presented a new physical model of the bowed string, which bases upon a waveguide string excited by a sophisticated elasto-plastic friction model. An accurate numerical implementation has been developed and applied to simulations of a bowed-string system.

The results presented in section 3.2 show that the model exhibits a physically consistent behavior, specifically the force versus velocity characteristics has a hysteretical behavior that has been observed in experiments on real instruments.

Preliminary comparisons with the plastic model by Smith and Woodhouse [3] show that the two models behave in qualitatively similar ways, although they have been derived under very different physical assumptions. Further work is required to develop quantitative comparisons between the two models. Specifically, non-linear identification techniques such as those described in [14] may be used in order to study whether parameters values of the elasto-

plastic model can be found such that the model behavior reproduce accurately that of the plastic model.

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